Sample: Electric Circuits - Analysing Sensor Circuits

Task 1 – Analysing the sensor circuit

(LO 3: 3.1 part)

The two sensors are the same, but it is suspected that they have slightly different characteristics. Two tests were carried out to find out how much voltage they give out and how much internal resistance they have. Figure 1 shows the equivalent circuit of each sensor, showing the internal voltage source (VS), the internal resistance (R1), a connecting resistor (R2) and a load resistor for the tests (R3).

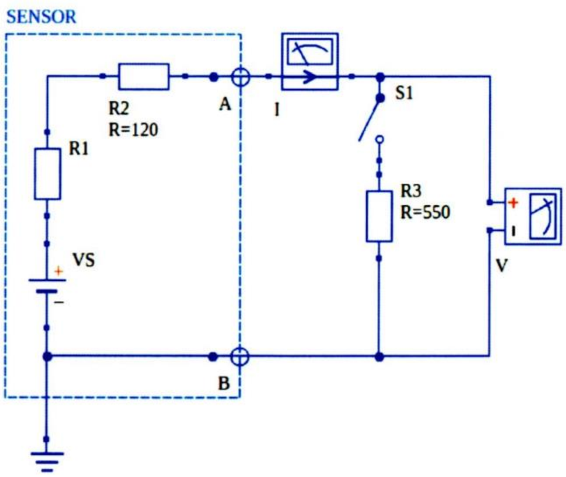

Figure 1: Sensor test circuit

The first test, with the switch open, gave the open-circuit voltage of the sensor. The second test, with the switch closed, used the load resistor to draw some current from the sensor and measure its voltage drop. Table 2 shows the results from both tests for both sensors.

sensor	switch open		switch closed	
	I (mA)	V (V)	I (mA)	V (V)
1	0	6.5	9.49	5.22
2	0	6.2	9.01	4.96

Table 2: Sensor test results

Use the results in Table 2, along with Ohm's law and Kirchhoff's laws, to determine the internal voltage source (VS), and the internal resistance (R1) to the nearest 1 Ω, for both sensors. (3.1 part)

Solution.

Consider the first test, for which the switch is open and $I = 0\,\text{mA}$. We see that the voltmeter is ideal (i.e. its internal resistance is equal to infinity) because $I = 0\,\text{mA}$. According the Kirchhoff's Voltage Law, write down the following equation:

$$IR1 + IR2 + V = VS \iff VS = V.$$

Thus, the internal voltage sources of the sensors are $VS_1 = 6.5\,V$ and $VS_2 = 6.2\,V$.

Consider the second test, for which the switch is closed. To find the resistance $R1$, we suppose that the ammeter is ideal (i.e. its internal resistance is equal to zero). We have in accordance with Kirchhoff's Voltage Law:

$$IR1 + IR2 + IR3 = VS \iff R1 = \frac{VS - I(R2 + R3)}{I} = \frac{VS}{I} - R2 - R3.$$

Find the internal resistance $R1_1$ of the first sensor:

$$R1_1 = \frac{VS_1}{I} - R2 - R3 = \frac{6.5\text{ V}}{9.49 \cdot 10^{-3}\text{ A}} - 120\,\Omega - 550\,\Omega = 14.932\,\Omega \approx 15\,\Omega.$$

Find the internal resistance $R1_2$ of the second sensor:

$$R1_2 = \frac{VS_2}{I} - R2 - R3 = \frac{6.2\text{ V}}{9.01 \cdot 10^{-3}\text{ A}} - 120\,\Omega - 550\,\Omega = 18.124\,\Omega \approx 18\,\Omega.$$

Task 2 – Analysing the effect of connecting the two sensors in parallel

(LO 3: 3.1 part, 3.2)

It was now decided to see if the sensors could be paralleled together, in order to supply more current. Figure 2 shows the equivalent circuit of the two sensors connected together in parallel.

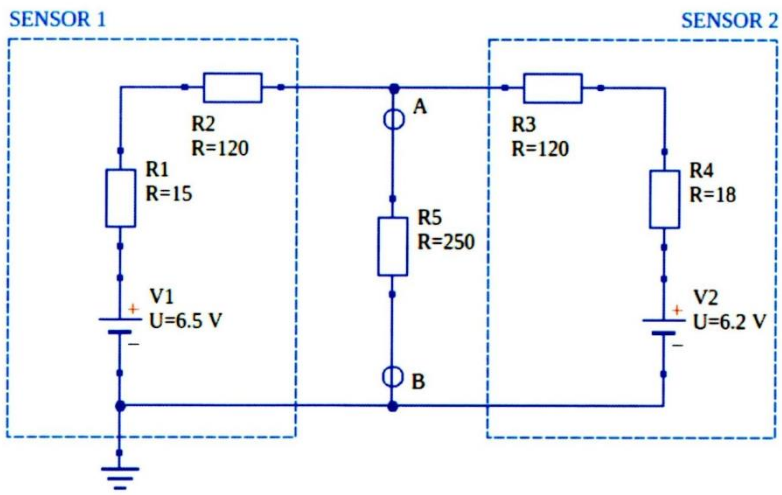

Figure 2: Sensors connected in parallel

First use Kirchhoff's laws and mesh analysis to determine the current supplied from each sensor, and the total current and voltage developed in the load resistor R5.

Next, create a Thevenin equivalent circuit of the circuit in Figure 2 (the output terminals are A-B and R5 is considered as the load resistor), and verify that this new circuit develops the same current and voltage in the 250 Ω load resistor as the original circuit in Figure 2.

Now use this Thevenin equivalent circuit to determine the current and voltage which would be developed in load resistors of (i) 10 Ω and (ii) 10 kΩ, which would be connected between terminals A and B in place of R5. What value load resistor would lead to maximum power being transferred from the sensors to the load?

(3.1 part, 3.2)

Solution.

Write the system of equations according Kirchhoff's Voltage Law (see Figure 2-1):

$$\begin{cases} I_1 R1 + I_1 R2 + I_3 R5 = V1; \\ I_2 R4 + I_2 R3 + I_3 R5 = V2. \end{cases}$$

Express the current I_3 through the mesh currents I_1 and I_2 using Kirchhoff's Current Law:

$$I_3 - I_1 - I_2 = 0 \Leftrightarrow I_3 = I_1 + I_2.$$

Plug I_3 into the system of equations:

$$\begin{cases} I_1 R1 + I_1 R2 + (I_1 + I_2) R5 = V1; \\ I_2 R4 + I_2 R3 + (I_1 + I_2) R5 = V2; \end{cases} \Leftrightarrow \begin{cases} I_1 (R1 + R2 + R5) + I_2 R5 = V1; \\ I_1 R5 + I_2 (R3 + R4 + R5) = V2. \end{cases}$$

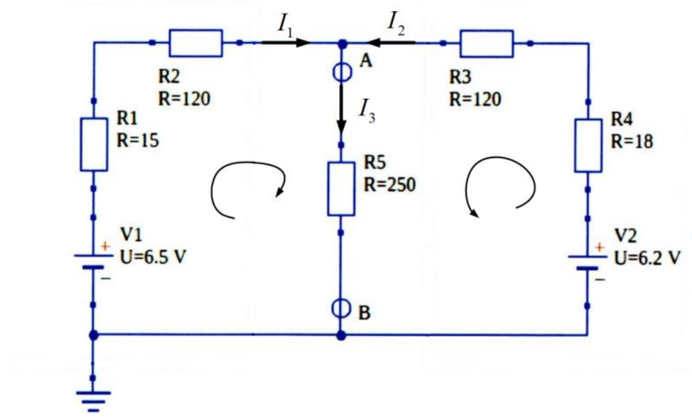

Figure 2-1

Plug the numerical values into the system of equation. We have:

$$\begin{cases} I_1(15+120+250)+250I_2=6.5; \\ 250I_1+I_2(120+18+250)=6.2; \end{cases} \Leftrightarrow \begin{cases} 385I_1+250I_2=6.5; \\ 250I_1+388I_2=6.2. \end{cases}$$

Solve the system by Cramer's Rule:

$$I_1=\frac{\begin{vmatrix} 6.5 & 250 \\ 6.2 & 388 \end{vmatrix}}{\begin{vmatrix} 385 & 250 \\ 250 & 388 \end{vmatrix}}=\frac{972}{86880}=0.0112\text{ A=11.2 mA};\ \ I_2=\frac{\begin{vmatrix} 385 & 6.5 \\ 250 & 6.2 \end{vmatrix}}{\begin{vmatrix} 385 & 250 \\ 250 & 388 \end{vmatrix}}=\frac{762}{86880}=0.0088\text{ A=8.8 mA}.$$

Find the current I_3 through the load resistor:

$$I_3=I_1+I_2=11.2\text{ mA}+8.8\text{ mA}=20\text{ mA}.$$

Draw the Thevenin equivalent circuit for the output terminals A-B as shown in Figure 2-2.

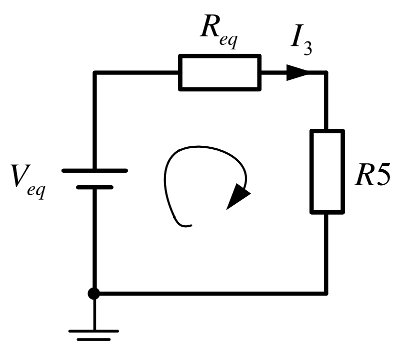

Figure 2-2

Find the equivalent voltage for the Thevenin circuit V_{eq}. This voltage is obtained at terminals A-B of the circuit with terminals A-B open circuited, as shown in Figure 2-3. According Kirchhoff's Voltage Law, write the following equation for the circuit shown in Figure 2-3:

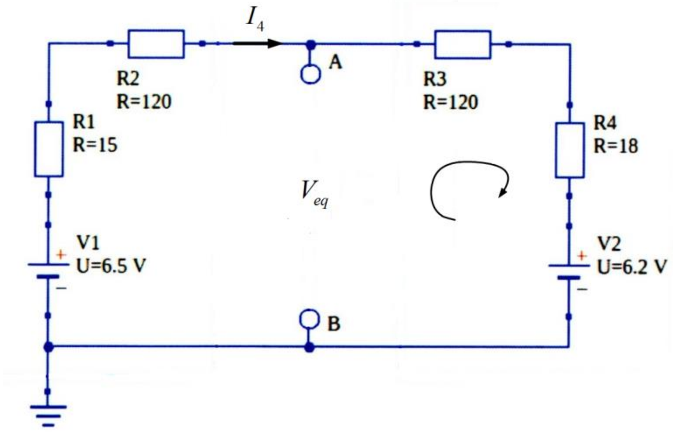

Figure 2-3

$$I_4R1 + I_4R2 + I_4R3 + I_4R4 = V1 - V2 \Leftrightarrow I_4 = \frac{V1 - V2}{R1 + R2 + R3 + R4}.$$

Use Kirchhoff's Voltage Law again to find the equivalent voltage V_{eq} :

$$V_{eq} = I_4R_3 + I_4R_4 + V2 = \frac{(V1 - V2)(R3 + R4)}{R1 + R2 + R3 + R4} + V2 = \frac{V1(R3 + R4) + V2(R1 + R2)}{R1 + R2 + R3 + R4} = 6.35\text{ V}.$$

Calculate the output current I_{AB} when the output terminals A and B are short circuited (see Figure 2-4). Using Kirchhoff's Voltage Law twice, we have:

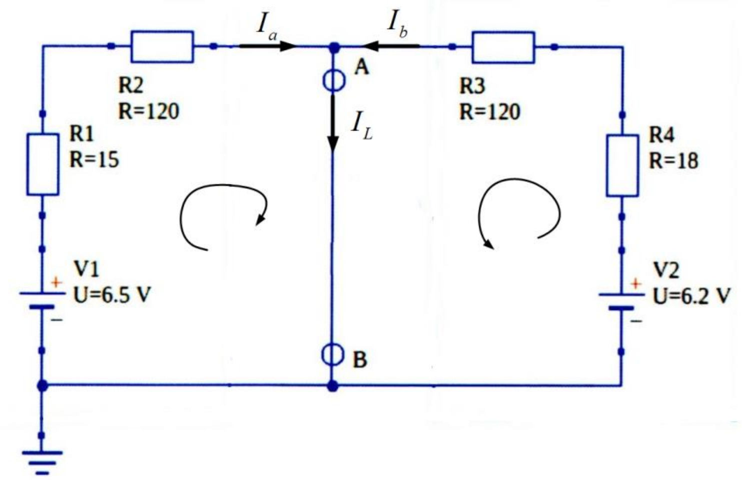

Figure 2-4

$$I_aR1 + I_aR2 = V1 \Leftrightarrow I_a = \frac{V1}{R1 + R2};\ \ I_bR3 + I_bR4 = V2 \Leftrightarrow I_b = \frac{V2}{R3 + R4}.$$

Express the current I_L according Kirchhoff's Current Law:

$$I_L = I_a + I_b = \frac{V1}{R1 + R2} + \frac{V2}{R3 + R4} = 93.1\text{ mA}.$$

Thus, the equivalent resistance R_{eq} for the Thevenin circuit (see Figure 2-2) is:

$$R_{eq} = \frac{V_{eq}}{I_L} = \frac{6.35\text{ V}}{9.31 \cdot 10^{-2}\text{ A}} = 68.2\,\Omega.$$

According to Ohm's Law, the current I_3 through the resistor $R5$ is given by (see Figure 2-2):

$$I_3 = \frac{V_{eq}}{R_{eq} + R_5} = \frac{6.35\text{ V}}{68.2\,\Omega + 250\,\Omega} = 20.0\text{ mA}.$$

As we can see, this result is the same as we got earlier.

Find I_3 for the load resistor $R_5 = 10\,\Omega$:

$$I_3 = \frac{V_{eq}}{R_{eq} + R_5} = \frac{6.35\text{ V}}{68.2\,\Omega + 10\,\Omega} = 81.2\text{ mA}.$$

Find I_3 for the load resistor $R_5 = 10\text{ k}\Omega$:

$$I_3 = \frac{V_{eq}}{R_{eq} + R_5} = \frac{6.35\text{ V}}{68.2\,\Omega + 10000\,\Omega} = 0.63\text{ mA}.$$

Find the power p_5 transferred to the load $R5$:

$$p_5 = I_3^2 R_5 = \frac{V_{eq}^2 R_5}{(R_{eq} + R_5)^2}.$$

We can consider p_5 as a function of resistance R_5. The function $p_5(R5)$ reaches its maximum when its derivative equals zero:

$$\frac{dp_5}{dR_5} = \frac{d}{dR_5}\left[\frac{V_{eq}^2 R_5}{(R_{eq} + R_5)^2}\right] = V_{eq}^2 \frac{(R_{eq} + R_5)^2 - 2(R_{eq} + R_5)R_5}{(R_{eq} + R_5)^4} = 0;\; R_{eq} + R_5 - 2R_5 = 0;\; R_5 = R_{eq} = 68.2\,\Omega.$$

Thus, when $R5 = 68.2\,\Omega$, the power transferred to the load $R5$ is maximal.