Sample: Integral Calculus - Area and Integral

Question 1

a) Consider the definite integral

$$\int_{-1}^{1} (x^3 + 3x^2)\,dx$$

i) State the antiderivative of the integral $f(x) = x^3 + 3x^2$

IMPORTANT: DO NOT add a "+ c". It is not necessary when finding an antiderivative to evaluate a definite integral.

Answer: $F(x) = \frac{x^4}{4} + x^3$.

ii) Use your answer from i) to evaluate the definite integral

$$\int_{-1}^{1} (x^3 + 3x^2)\,dx.$$

Answer: $\int_{-1}^{1} (x^3 + 3x^2)\,dx = 2$.

b) Consider the definite integral

$$\int_{0}^{1} 6x^2(x^3 + 1)^4\,dx.$$

i) State the appropriate substitution which can be used to evaluate this integral.

Answer: $u = (x^3 + 1)$.

ii) State the values of the integration bounds in the new variable

Answer: $x = 0 \Rightarrow u = 1$

$x = 1 \Rightarrow u = 2$

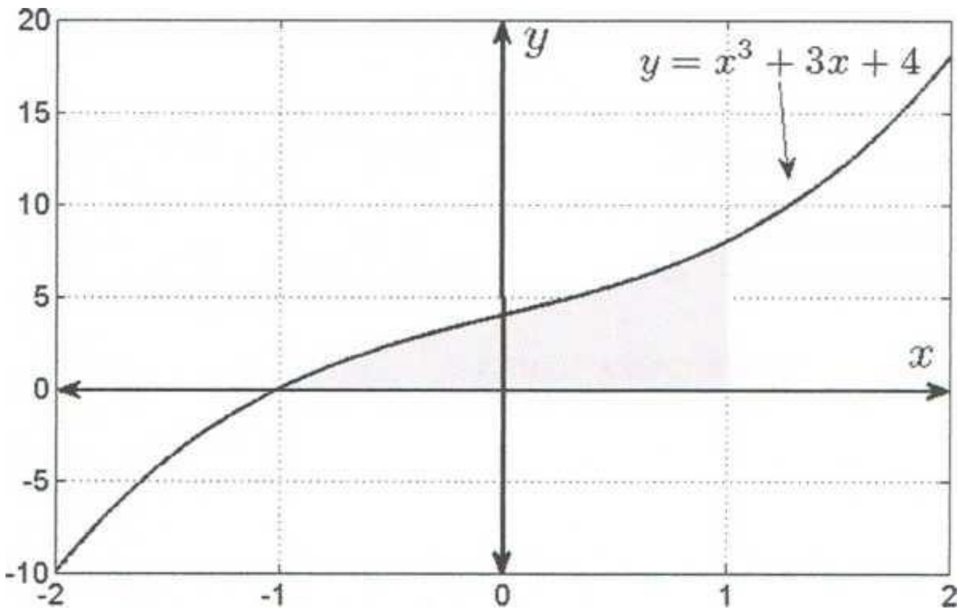
iii) Use your answers from i) and ii) to evaluate the definite integral

$$\int_{0}^{1} 6x^2(x^3 + 1)^4\,dx.$$

Answer: $\int_{0}^{1} 6x^2(x^3 + 1)^4\,dx = \frac{62}{5}$.

Question 2

Consider the shaded region in the figure below.



a) The area of the shaded region can be expressed in the form of a definite integral

$$\int_a^b f(x)\,dx$$

Specify the values of a and b, and the expression for f.

Answer:

$$a = -1$$

$$b = 1$$

$$f(x) = x^3 + 3x + 4$$

b) State the antiderivative of the integrand in a).

IMPORTANT: DO NOT add a "+ c". It is not necessary when finding an antiderivative to evaluate a definite integral.

Answer:

$$F(x) = \frac{x^4}{4} + \frac{3x^2}{2} + 4x.$$

c) Use your answers from a) and b) to determine the area of the shaded region.

Answer: $Area = 8.$

Question 3

Differentiate the following functions with respect to x.

a) $f(x) = \int_0^x \frac{1}{t+1} dt.$

Answer: $\frac{df}{dx} = \frac{1}{x+1}.$

b) $f(x) = \int_1^{2x^2} \frac{1}{t+1} dt.$

Answer: $\frac{df}{dx} = \frac{4x}{2x^2+1}.$