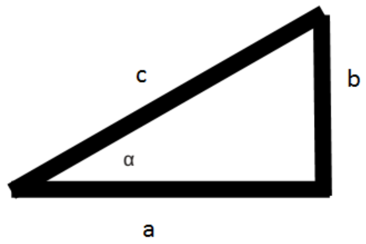
Sample: Trigonometry - Trigonometric Identities

1. Evaluate $cos\left(arctan\left(sin\left(arctan(\sqrt{2}/2)\right)\right)\right)$



a) let $\alpha = arctan(\sqrt{2}/2)$

b) this means that $tan(\alpha) = \sqrt{2}/2$

c) see photo, then $b = \sqrt{2}, a = 2$ using Pythagorean theorem give us

$c = \sqrt{6}$

d)find $sin(\alpha) = b/c = \sqrt{2}/\sqrt{6} = \sqrt{2/6} = \sqrt{1/3}$

e)find $arctan\left(\sqrt{1/3}\right) = \pi/6$

f) find $cos(\pi/6) = \sqrt{3}/2$

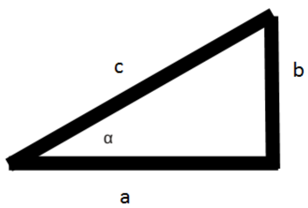
Answer:$\sqrt{3}/2$

2. Evaluate $sin(arccos(3/5) - arctan(7/13))$

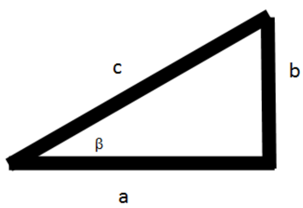
a) general formula : $sin(\alpha - \beta) = sin(\alpha)cos(\beta) - cos(\alpha)sin(\beta)$

b) let $\alpha = arccos(3/5)$, $\beta = arctan(7/13)$

c) Consider a right triangle for angle α: a=3, c=5. Using Pythagorean theorem give us



d)Consider a right triangle for angle β: a=13, b=7.Using Pythagorean theorem give us $c = \sqrt{218}$



e)Calculate $sin(\alpha) = 4/5\, , cos(\alpha) = 3/5, sin(\beta) = 7/\sqrt{218}\, , cos(\beta) = 13/\sqrt{218}$

f) Calculate $sin(\alpha - \beta) = \frac{\frac{4}{5}*13}{\sqrt{218}} - \frac{\frac{3}{5}*7}{\sqrt{218}} = \frac{31}{5*\sqrt{218}}$

Answer: $\frac{31}{5*\sqrt{218}}$;

3.Evaluate $sin\left(arcsin\left(\frac{1}{3}\right) + arcsin\left(\frac{1}{4}\right)\right)$

a)general formula:$sin(\alpha + \beta) = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$

b)let $\alpha = \arcsin(\frac{1}{3})$,$\beta = \arcsin(\frac{1}{4})$

c)Know that: $sin\big(arcsin(m)\big) = m$, so $sin(\alpha) = \frac{1}{3}, sin(\beta) = \frac{1}{4}$

d)Know that:$sin(x)^2 + cos(x)^2 = 1$, so $cos(\alpha) = \sqrt{1 - \frac{1}{3^2}} = \frac{2*\sqrt{2}}{3}$, $cos(\beta) = \sqrt{1 - \frac{1}{4^2}} = \frac{\sqrt{15}}{4}$

e) use a general formula : $sin(\alpha+\beta)=\frac{\left(\frac{1}{3}*\sqrt{15}\right)}{4}+\frac{\frac{2*\sqrt{2}}{3}*1}{4}=\frac{\sqrt{15}+2*\sqrt{2}}{12}$

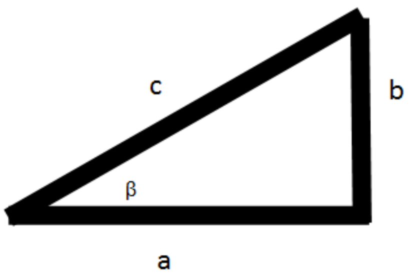
Answer: $\frac{\sqrt{15}+2*\sqrt{2}}{12}$;

4. Evaluate $\tan(\frac{\pi}{4}+\arcsin\left(\frac{5}{13}\right))$

a) a general formula : $tan(\alpha+\beta)=\frac{tan(\alpha)+tan(\beta)}{1-tan(\alpha)*tan(\beta)}$

b) let $\alpha=\frac{\pi}{4},\beta=arcsin(5/13)$

c) $tan(\pi/4)=1$



d) see photo b=5, c=13 using Pythagorean theorem give us a= 12

e) $tan(\beta)=\frac{5}{12}$

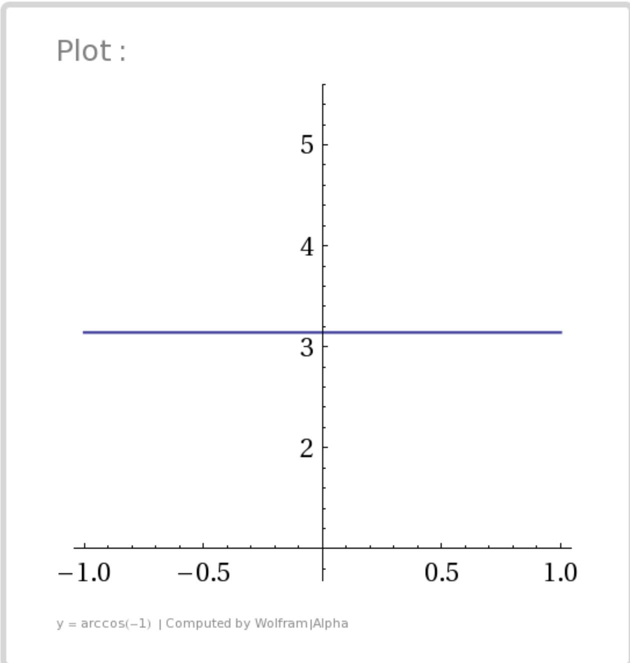
f) Using formula from a), we calculate answer:

$tan(\alpha+\beta)=\frac{1+\frac{5}{12}}{1-\frac{5}{12}}=\frac{12+5}{12-5}=\frac{17}{7}$

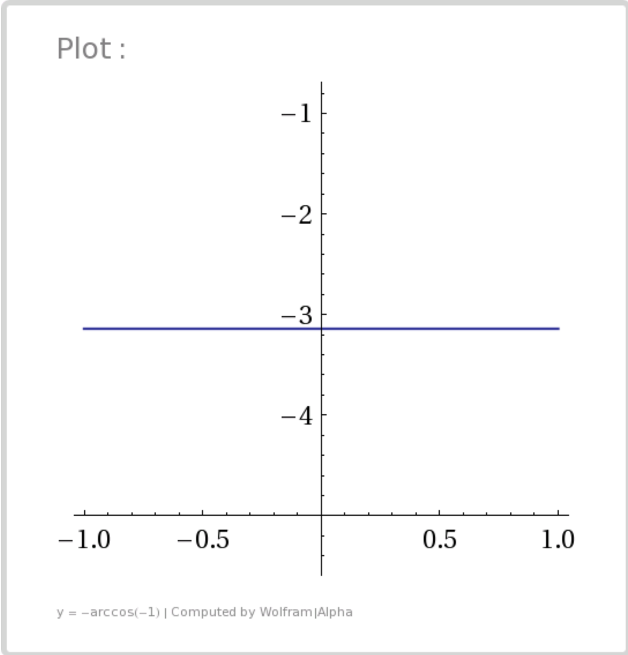
Answer: $\frac{17}{7}$;

5. Graph the following functions:

a) $y=\arccos(-1)$
General formula: $\arccos(-x)=\pi-\arccos(x)$, so in our case :
$y=\pi$



b) $y = -\arccos(-1)$ Use result of item a we have: $y = -\pi$



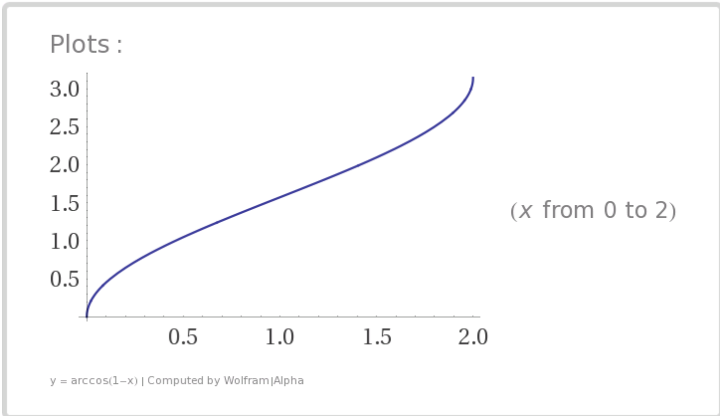
c)$y = \arccos(1 - x)$

in general domain of $\arccos(x): -1 \leq x \leq 1$, in our case:

$-1 \leq 1 - x \leq 1;$

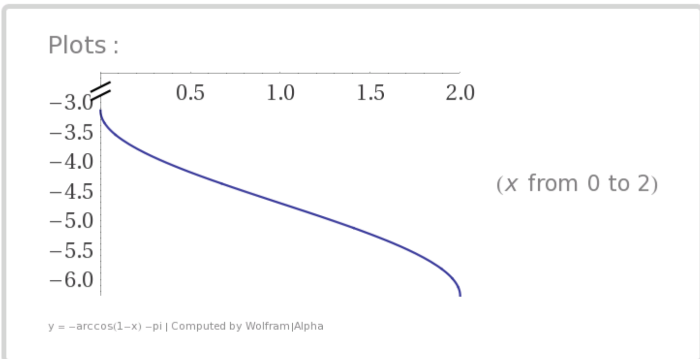
$1 \geq x - 1 \geq -1;$

$2 \geq x \geq 0;$



d)$y = -\arccos(1 - x) - \pi$

Use result from item c: make some transformation: first: reflection in the x-axis,second : the graph is shifted π places down.



6. Simplify the expressions:

General formula for item a) $\sin(\alpha) * \cos(\beta) = \frac{\sin(\frac{\alpha-\beta}{2}) + \sin(\frac{\alpha+\beta}{2})}{2}$

General formulas for item b) $\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right) * \cos(\frac{\alpha-\beta}{2})$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right) * \cos(\frac{\alpha-\beta}{2})$$

a) $sin\left(\frac{7*\pi}{24}\right) * cos\left(\frac{\pi}{24}\right) = \frac{sin(\frac{\pi}{4})+sin(\frac{\pi}{3})}{2} = \frac{\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{2}+\sqrt{3}}{4}$

b) $\frac{sin(3x)+sin(5x)}{cos(3x)+cos(5x)} = \frac{2*sin((3x+5x)/2)cos((5x-3x)/2)}{2*cos((5x+3x)/2)cos((5x-3x)/2)} = tan(4x)$

7. If $\cos(\alpha) = \frac{2}{\sqrt{5}}$ $\left(\frac{3\pi}{2} < \alpha < 2\pi\right)$ and $\sin(\beta) = \frac{4}{5}$ $(\frac{\pi}{2} < \beta < \pi)$ complete the following:

a) $\sin(\alpha - \beta)$

We know that: $\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$;

$\sin(\alpha) = -\sqrt{1 - cos^2(\alpha)} = -\frac{1}{\sqrt{5}}$, $\cos(\beta) = -\sqrt{1 - sin^2(\beta)} = -\frac{3}{5}$, according to the scope of α and β;

$\sin(\alpha - \beta) = \left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{3}{5}\right) - \left(\frac{2}{\sqrt{5}}\right)\left(\frac{4}{5}\right) = -\frac{1}{\sqrt{5}}$;

Answer : $-\frac{1}{\sqrt{5}}$;

b) $\cos\left(\frac{\beta}{2}\right)$

Known that: $\cos(\beta) = 2cos^2\left(\frac{\beta}{2}\right) - 1 \rightarrow \cos\left(\frac{\beta}{2}\right) = \sqrt{\frac{\cos(\beta)+1}{2}}$, because $\left(\frac{\pi}{4} < \frac{\beta}{2} < \frac{\pi}{2}\right)$;

$\cos\left(\frac{\beta}{2}\right) = \sqrt{\frac{\left(-\frac{3}{5}\right)+1}{2}} = \frac{1}{\sqrt{5}}$;

Answer : $\frac{1}{\sqrt{5}}$;

c) $\tan(2\alpha) + \tan(2\beta)$

Known that: $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$ and $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$;

$\tan(\alpha) = \frac{-\frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}} = -\frac{1}{2}$, $\tan(\beta) = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$;

$\tan(2\alpha) = \frac{2\left(-\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = -\frac{4}{3}$, $\tan(2\beta) = \frac{2\left(-\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{24}{7}$

$\tan(2\alpha) + \tan(2\beta) = -\frac{4}{3} + \frac{24}{7} = \frac{44}{21}$;

Answer : $\frac{44}{21}$.